1. Determine whether these statements are true or false.

a) $\emptyset \in \{\emptyset\}$
b) $\emptyset \in \{\emptyset, \{\emptyset\}\}$
c) $\{\emptyset\} \in \{\emptyset\}$
d) $\{\emptyset\} \in \{\{\emptyset\}\}$
e) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$
f) $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$
g) $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$

2. What is the cardinality of each of these sets?

a) $\emptyset$
b) $\{\emptyset\}$
c) $\{\emptyset, \{\emptyset\} \}$
d) $\{\emptyset, \{\emptyset\} ,\{\emptyset,\{\emptyset\}\}\}$

3. Determine whether each of these sets is the power set of a set, where *a* and *b* are distinct elements

a) $\emptyset$
b) $\{\emptyset, \{a\}\}$
c) $\{\emptyset, \{a\}, \{\emptyset, a\}\}$
d) $\{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

4. Explain why $(A \times B) \times (C \times D)$ and $A \times (B \times C) \times D$ are not the same

5. Suppose that $\mathrm{A} \times B = \emptyset$, where A and B are sets, what can you conclude?

6. Let $A = \{a, b, c, d, e\}$ and $B = \{a, b, c, d, e, f, g, h\}$. Find

a) $A \cup B$
b) $A \cap B$
c) $A - B$
d) $B - A$

7. Show that if A and B are sets, then $(A \cap B) \cup (A \cap \bar{B}) = A$.

8. Show that $A \oplus B = (A \cup B) - (A \cap B)$

9. Find the domain and range of these functions

b) The function that assigns to each positive integer its largest decimal digit

c) The function that assigns to a bit string the number if ones minus the number of zeros in the string

e) The function that assigns to a bit string the longest string of ones in the string

10. Find these values:

a) $\lfloor 1.1 \rfloor$

b) $\lceil 1.1 \rceil$

c) $\lfloor -0.1 \rfloor$

d) $\lceil -0.1 \rceil$

e) $\lceil 2.99 \rceil$

f) $\lceil -2.99 \rceil$

g) $\left\lfloor \frac{1}{2} + \left\lceil \frac{1}{2} \right\rceil \right\rfloor$

h) $\left\lceil \left\lfloor \frac{1}{2} \right\rfloor + \left\lceil \frac{1}{2} \right\rceil + \frac{1}{2} \right\rceil$

11. Determine whether each of these functions from $\mathbf{Z}$ to $\mathbf{Z}$ is one to one.

a) $f(n) = n - 1$

b) $f(n) = n^2 + 1$

c) $f(n) = n^3$

d) $f(n) = \lceil n/2 \rceil$

12. Determine whether $f: \mathbf{Z} \times \mathbf{Z} \to \mathbf{Z}$ is onto if

a) $f(m, n) = 2m - n$

b) $f(m, n) = m^2 - n^2$

c) $f(m, n) = m + n + 1$

d) $f(m, n) = |m| - |n|$

e) $f(m, n) = m^2 - 4$

13. Determine whether each of these functions is a bijection from $\mathbf{R}$ to $\mathbf{R}$

a) $f(x) = -3x + 4$

b) $f(x) = -3x^2 + 7$

c) $f(x) = (x + 1)/(x + 2)$

d) $f(x) = x^5 + 1$

14. Let $f(x) = ax + b$ and $g(x) = cx + d$, where $a,\ b,\ c,$ and d are constaints. Determine for which constants $a,\ b,\ c,$ and d it is true that $f \circ g = g \circ f$.

15. Suppose that f is a function from A to B, where A and B are finite sets with $|A| = |B|$
Show that f is one-to-one iff it is onto

16. Let f and g be functions from $\{1,\ 2,\ 3,\ 4\}$ to $\{a,\ b,\ c,\ d\}$ and from $\{a,\ b,\ c,\ d\}$ to $\{1,\ 2,\ 3,\ 4\}$ respectively, such that $f(1) = d,\ f(2) = c,\ f(3) = a,\ f(4) = b$ and $g(a) = 2,\ g(b) = 1,\ g(c) = 3,$ $g(d) = 2$

a) Is f one-to-one? Is g one-to-one?

b) Is f onto? Is g onto?

c) Does either f or g have an inverse?

17. How many bytes are required to encode n bits of data where n equals

a) 4?

b) 10?

c) 500?

d) 3000?

18. Prove $\lceil \lceil x/2 \rceil /2 \rceil = \lceil x/4 \rceil$ for all real number x

19. Find at least three different sequences beginning with the terms 3, 5, 7 whose terms are generated by a simple formula or rule.

20. For each of these lists of integers, provide a simple formula or rule that generates the terms of an integer sequence that begins with the given list. Assuming that your formula or rule is correct, determine the next three terms of the sequence.

a) 3, 6, 11, 18, 27, 38, 51, 66, 83, 102, …

d) 1, 2, 2, 2, 3, 3, 3, 3, 3, 5, 5, 5, 5, 5, 5, 5, …

e) 0, 2, 8, 26, 80, 242, 728, 2186, 6560, 19682, …

21. Determine whether each of these sets is countable or uncountable. For those that are countable, exhibit a one-to-one correspondence between the set of natural numbers and that set.

a) the integers greater than 10

d) integers that are multiples of 10